**EMTH211 Assignment Cover Sheet**

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Tutorial Group: (09) -- Friday 4pm

**Introduction:**

Within this report I will discuss the application of Markov chains; and their specific application to modelling a visitor at a theme park, Leslie Models; and their specific application to modelling the population growth of mosquito’s. Along with the use of Gershgorin’s Theorem to locate eigenvalues efficiently.

**Q1:**

1. The general equation to compute the probability after k steps, where k is the number of events occurring is: . where A is the transition matrix, D is the diagonalized matrix and P is the eigenvalue matrix corresponding to D. This decomposition is technically not needed, however for large values it is much faster computationally and therefore necessary for when A is a large matrix and k approaches infinity (long-term distribution). To compute the long-term distribution, k was set to 1 million as a good approximation to infinity. This gave the probability shown below in Table 1.

Table 1: Long-term distribution of which attraction visitors visit.

|  |  |
| --- | --- |
| Attraction | Probability (4 s.f.) |
| A | 9.589% |
| B | 13.70% |
| C | 13.70% |
| D | 17.80% |
| E | 13.70% |
| F | 17.80% |
| G | 13.70% |

1. The probability that at any given time a visitor will go to the same attraction at any given time is the probability that a visitor at an attraction will visit the same attraction two more times, that is (), assuming this is at any given time point where they are already at an attraction , and therefore the probability would simply be . Where the probability for each attraction is the diagonal entry of the matrix that corresponds to the attraction. However, for the long-term will be equal to the probability calculated in 1a and shown in Table 1. Multiplying the Table 1 by the of each attraction gives the following Table 2.

Table 2: Long-term probability of a visitor visiting an attraction 3 time in a row.

|  |  |
| --- | --- |
| Attraction | Probability (4 s.f.) |
| A | 0.1957% |
| B | 0.1370% |
| C | 0.1370% |
| D | 0.1054% |
| E | 0.1370% |
| F | 0.1054% |
| G | 0.1370% |

1. There is a lot of limitations when it comes to modelling the theme park visitor’s behavior using Markov Chains, this is mainly because our probability does not change based on events within the theme park. A few examples of these include queues; a visitor would most likely have a lower probability of visiting an attraction if the line for the attraction is long or exposure; a visitor may have a higher probability of going onto an attraction because they can see others enjoying it or the advertisement for the attraction is greater than other adjacent attractions. Another reason the Markov Chain has limitation is because it does not consider what the visitor has already done. Two main examples of this can be seen. The first one being repeated attractions; the probability a visitor goes on the same attraction again will most likely lower the more times he has been on it, especially in repeated succession. The second example is the idea that many visitors may want to “complete” the theme park before going on attractions for a second time, so they can find the best/most enjoyable attractions.

**Q2:**

1. Chart

   Description automatically generated with medium confidenceLeslie models are used to model the population growth/decline of a species. To do this the survival rate and birth rate at specific age ranges is required. The birth rate is then used as the first row of the Leslie matrix and the remaining rows have diagonals that correspond to the survival rate of that age group. The plot of 50 iterations is shown below in Figure 1, each age group is highlighted as a different color.

Figure 1: Leslie Matrix in python using NumPy.

1. After 50 iterations of Leslie’s Method, the mosquito population has not yet hit a plateau as shown on the graph and has therefore not yet approached the theoretical long-term distribution. This is seen as there is a 30.44% increase between the 49th and 50th iterations. This means the population has yet to plateau.
2. The dominant eigenvalue for the Leslie matrix is . Since the eigenvector is greater than 1 that means that the population is naturally going to grow, and we can harvest some of the mosquitos at birth. Using the equation , where h is the harvest rate of the mosquito eggs and R is the net reproduction rate (3.431). This means we can harvest 70.856% of the mosquito eggs. This number is used to create a harvest matrix with a value of 0.70856 in the position. It is then subtracted from the identity matrix and multiplied with the original Leslie matrix we had. This gives effective birth rates of 29.114 across all ages instead of the original 100. As proof of our working we can use la.eig() on NumPy to see that the dominant eigenvector is 1 and therefore the population of mosquitos will reach a plateau.

**Q3:**

1. Row based Gerschgorin discs are:

Column based Gerchgorin discs are:

Chart, bubble chart

Description automatically generatedPlots of these graphs are shown below in Figure 2 and 3, where the Y axis is the imaginary plane, and the X axis is the real plane.

Chart, bubble chart

Description automatically generatedFigure 2: Column Based Gerchgorin discs

Figure 3: Row based Gerchgorin discs

1. Gercshgorin’s Theorem has been used to plot discs on the real and imaginary planes, where the area that they contain is the values that an eigenvalue must be. Plotting both the column and row discs restricts the area further to a location that must be within both discs. Since neither Column nor Row Gerchgorin discs encapsulate 0, the matrix A must be invertible as no eigenvalue can be 0. The eigenvalues for this matrix are 3.25, 1.99, -5.24, -6.

**Conclusion:**

In this report I showed how to apply and use Markov Chains, Leslie Method and Gerchgorin discs. This was done through understanding a visitor at an amusement park. The population growth of mosquitos and how to find estimation for the eigenvalues of a matrix.